

TEST FLIGHT PROBLEM SET: Problem 10

Problem

To give a family of intervals $A_n, n=1, 2, \dots$
such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n$ consists
of a single real number

Solution

Consider the intervals $A_n = [0, 2^{-n}]$

Clearly $A_{n+1} \subset A_n$ for all n

Consider an arbitrary $\delta > 0$

We can always find a number N so big that $2^{-N} < \delta$

$$2^{-N} < \delta \Rightarrow \delta \notin A_N \Rightarrow \delta \notin \bigcap_{n=1}^{\infty} A_n$$

~~Since~~ Since δ is arbitrary, this proves that $\bigcap_{n=1}^{\infty} A_n$ contains
no element > 0

Note that $0 \in A_n$ for all $n \Rightarrow 0 \in \bigcap_{n=1}^{\infty} A_n$

This completes the proof that the family of intervals

$[0, 2^{-n}]$ for $n=1, 2, \dots$ satisfies the conditions of the problem