

TEST FLIGHT PROBLEM SET: Problem 9

Problem:

To give an example of a family of intervals $A_1, A_2, \dots, A_n, \dots$ such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \emptyset$

Solution:

Consider the intervals $A_n = (0, 2^{-n})$ for $n = 1, 2, \dots$

Clearly $A_{n+1} \subset A_n$ for all n

Take an arbitrary $\delta > 0$

We can always find some number N so big that $2^{-N} < \delta$

$$2^{-N} < \delta \Rightarrow \delta \notin A_N \Rightarrow \delta \notin \bigcap_{n=1}^{\infty} A_n$$

Since δ is arbitrary, there is no $x > 0 \notin \bigcap_{n=1}^{\infty} A_n$

Clearly 0 is not an element of A_1 , and therefore $0 \notin \bigcap_{n=1}^{\infty} A_n$

This completes the proof that $\bigcap_{n=1}^{\infty} A_n = \emptyset$

It follows that the intervals $(0, 2^{-n})$ meet the conditions of the problem