

TEST FLIGHT PROBLEM SET : Problem 8

To prove:

If the sequence $\{a_n\}_{n=0}^{\infty}$ tends to the limit L , then the sequence $\{Ma_n\}_{n=0}^{\infty}$, where M is any fixed number > 0 , tends to the limit ML

Proof:

Take an arbitrary $\epsilon > 0$ and an arbitrary $M > 0$

By the definition of the limit of $\{a_n\}_{n=0}^{\infty}$

$$\exists N \text{ such that } |a_n - L| < \frac{\epsilon}{M} \quad \forall n > N$$

By arithmetic $|a_n - L| < \frac{\epsilon}{M} \Rightarrow M|a_n - L| < \epsilon$ ~~$M|a_n - L| < \epsilon$~~

$$\Rightarrow |Ma_n - ML| < \epsilon$$

It follows that $\exists N$ such that $|Ma_n - ML| < \epsilon \quad \forall n > N$

By the definition of the limit of $\{Ma_n\}_{n=0}^{\infty}$ this completes the proof.