

Test Flight Problem Set 7

Proposition: For any natural number n , $2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 2$

Proof by induction

Let $f(n) = 2 + 2^2 + 2^3 + \dots + 2^n$

$$f(1) = 2 = 2^2 - 2$$

$f(n+1) = f(n) + 2^{n+1}$ by the definition of $f(n)$

By the induction hypothesis $f(n) = 2^{n+1} - 2$

Substituting for $f(n)$ we have:

$$f(n+1) = 2^{n+1} - 2 + 2^{n+1} = 2 \cdot 2^{n+1} - 2 = 2^{n+2} - 2$$

This completes the proof